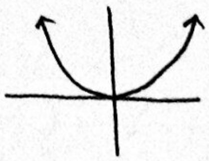
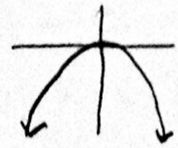
Lesson 19: Concavity and the Second Derivative Test

Ex

$y=x^2 \longrightarrow y'=2x \longrightarrow y''=2$



$y=-x^2 \longrightarrow y'=-2x \longrightarrow y''=-2$



If $y''>0$, then y is concave up (cu)

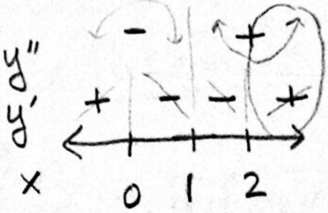
If $y''<0$, then y is concave down (cD)

Ex 1 Find the intervals where $y=x^3-3x^2$ is concave up/down.

$y'=3x^2-6x$

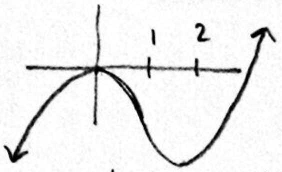
$y''=6x-6 \overset{set}{=} 0$

$x=1$



CD $(-\infty,1)$

CU $(1,\infty)$



← more on this in lesson 23

On which interval(s) is y cu and increasing? $(2,\infty)$

$\downarrow$ $y''>0$ ↳ $y'>0$

Def When f(x) changes concavity, we have an inflection point (IP).

Ex 1 IP at $(1, 1^3-3(1)^2) = \boxed{(1,-2)}$

↑ plug into y

Second Derivative Test

① Find CV's ($f'(c)=0$ or DNE, and $f(c)$ is defined)

② Find $f''(x)$.

③ If $f''(c)>0$: relative minimum at $x=c$.

If $f''(c)<0$: relative maximum at $x=c$.

If $f''(c)=0$ or DNE: ?? use 1st derivative test.

Ex 2 Find the relative extrema of $f(x)=x^5+5x^2$.

$f'(x)=5x^4+10x \overset{\text{set}}{=} 0$

$5x(x^3+2)=0$

$f''(x)=20x^3+10$

CV's: $x=0,\ \sqrt[3]{-2}$

at $x=0$: $f''(0)=20(0)^3+10>0$ $\boxed{\text{rel min at } x=0}$

at $x=\sqrt[3]{-2}$: $f''(\sqrt[3]{-2})=20(\sqrt[3]{-2})^3+10$

$=20(-2)+10<0$ $\boxed{\text{rel max at } x=\sqrt[3]{-2}}$